Scalar GW detection with a hollow spherical antenna

E.Coccia, F. Fucito and M.Salvino
Dipartimento di Fisica, Università di Roma "Tor Vergata" and
INFN Sezione di Roma Tor Vergata, Via Ricerca Scientifica 1, 00133 Roma, Italy

J. A. Lobo

Departament de Física Fonamental, Universitat de Barcelona, Spain

We study the response and cross sections for the absorption of GW energy in a Jordan-Brans-Dicke theory by a resonant mass detector shaped as a hollow sphere.

I. INTRODUCTION

It seems reasonable to predict that the new gravitational wave (GW) detectors now under construction, once operating at the maximum of their sensitivity, will be able to detect GWs. Will it be possible to use these future measurements to try to gain information on which is the theory of gravity at low energies? There are no particular reasons, in fact, why GW must be of spin two. In reality, many theories of gravity can be built which contain scalars and vectors. These theories are mathematically well founded. String theory, in particular, is believed to be consistent also as a quantum description of gravity. The predictions of these theories must then be checked against available experimental data. This forces the couplings and masses present in the Lagrangian to take values in well defined domains. See [1] for a more detailed exposition. Once detected, one can also attempt to use GWs as a means to further constrain this picture. It seems relevant to try to develop the theory to the point where it can profit from new experimental insights. For these reasons, it has been analysed in great detail in reference [2] —see also [3,4] the interaction and cross section of an elastic massive sphere with scalar waves.

An appealing variant of the massive sphere is a hollow sphere [5]. The latter has the remarkable property that it enables the detector to monitor GW signals in a significantly lower frequency range —down to about 200 Hz—than its massive counterpart for comparable sphere masses. This can be considered a positive advantage for a future world wide network of GW detectors, as the sensitivity range of such antenna overlaps with that of the large scale interferometers, now in a rather advanced state of construction [6,7]. Moreover it appears technically easier to fabricate in large dimensions [5].

A hollow sphere obviously has the same symmetry of the massive one, so the general structure of its *normal modes* of vibration is very similar in both [5]. In particular, the hollow sphere is very well adapted to sense and monitor the presence of scalar modes in the incoming GW signal. In this paper we extend the analysis of the response of a hollow sphere [5], to include scalar excitations.

In section II we briefly review the normal mode algebra of the hollow sphere; then in section III we calculate the scalar cross sections for the absorption of GW energy in scalar modes, and in section IV we assess the detectability of a few interesting sources on the assumption that they behave as Jordan-Brans-Dicke emitters [8,9] of GWs. Finally, section V is devoted to a summary of conclusions.

II. REVIEW OF HOLLOW SPHERE NORMAL MODES

This section contains some review material which is included essentially to fix the notation and to ease the reading of the ensuing sections. The eigenmode equation for a three-dimensional elastic solid is the following:

$$\nabla^2 \mathbf{s} + (1 + \lambda/\mu) \, \nabla(\nabla \cdot \mathbf{s}) = -k^2 \mathbf{s} \,\,, \qquad \left(k^2 \equiv \varrho \omega^2/\mu\right), \tag{2.1}$$

as described in standard textbooks, such as [10,11]. λ and μ are the material's Lamé coefficients, ρ is the material density and ω is the angular frequency.

The equation must be solved subject to the boundary conditions that the solid is to be free from tensions and/or tractions. In the case of a hollow sphere, we have two boundaries given by the outer and the inner surfaces of the solid itself. We use the notation a for the inner radius, and R for the outer radius. The boundary conditions are thus expressed by

$$\sigma_{ij}n_j = 0$$
 at $r = R$ and at $r = a$ $(R \ge a \ge 0)$, (2.2)

where σ_{ij} is the stress tensor, and is given by [11]

$$\sigma_{ij} = \lambda u_{k,k} \,\delta_{ij} + 2 \,\mu \,u_{(i,j)}. \tag{2.3}$$

where \mathbf{n} is the unit, outward pointing normal vector.

The general solution to equation (2.1) is a linear superposition of a longitudinal vector field and two transverse vector fields, i.e.,

$$\mathbf{s}(r,\vartheta,\phi) = \frac{C_{\mathrm{l}}}{q} \mathbf{s}_{\mathrm{l}} + \frac{C_{\mathrm{t}}}{k} \mathbf{s}_{\mathrm{t}} + C_{\mathrm{t}'} \mathbf{s}_{\mathrm{t}'}$$
(2.4)

where $C_{\rm l}$, $C_{\rm t}$ and $C_{\rm t'}$ are constant coefficients, and

$$\mathbf{s}_{l}(r,\vartheta,\phi) = \frac{dh_{l}(qr,E)}{dr} Y_{lm} \mathbf{n} - \frac{h_{l}(qr,E)}{r} i\mathbf{n} \times \mathbf{L} Y_{lm}$$
(2.5a)

$$\mathbf{s}_{t}\left(r,\vartheta,\phi\right) = -l\left(l+1\right)\frac{h_{l}\left(kr,F\right)}{r}Y_{lm}\mathbf{n} + \left[\frac{h_{l}\left(kr,F\right)}{r} + \frac{dh_{l}\left(kr,F\right)}{dr}\right]i\mathbf{n} \times \mathbf{L}Y_{lm}$$
(2.5b)

$$\mathbf{s}_{t'}(r, \theta, \phi) = h_l(kr, F) i \mathbf{L} Y_{lm} \tag{2.5c}$$

with E and F also arbitrary constants,

$$q^2 \equiv k^2 \frac{\mu}{\lambda + \mu} = \frac{\varrho_0 \omega^2}{\lambda + \mu} \tag{2.6}$$

and

$$h_l(z, A) \equiv j_l(z) + A y_l(z) \tag{2.7}$$

 $j_l \ y_l$ are spherical Bessel functions [12]:

$$j_l(z) = z^l \left(-\frac{1}{z} \frac{d}{dz} \right)^l \frac{\sin z}{z} \tag{2.8a}$$

$$y_l(z) = -z^l \left(-\frac{1}{z} \frac{d}{dz} \right)^l \frac{\cos z}{z} \tag{2.8b}$$

Finally, \mathbf{L} is the angular momentum operator

$$\mathbf{L} \equiv -i\,\mathbf{x} \times \nabla \tag{2.9}$$

The boundary conditions (2.2) must now be imposed on the generic solution to equations (2.1). After some rather heavy algebra it is finally found that there are two families of eigenmodes, the *toroidal* (purely rotational) and the *spheroidal*. Only the latter couple to GWs [13], so we shall be interested exclusively in them. The form of the associated wavefunctions is

$$\mathbf{s}_{nlm}^{S}(r, \vartheta, \phi) = A_{nl}(r) Y_{lm}(\vartheta, \phi) \mathbf{n} - B_{nl}(r) i \mathbf{n} \times \mathbf{L} Y_{lm}(\vartheta, \phi)$$
(2.10)

where the radial functions $A_{nl}(r)$ and $B_{nl}(r)$ have rather complicated expressions:

$$A_{nl}(r) = C(nl) \left[\frac{1}{q_{nl}^S} \frac{d}{dr} j_l(q_{nl}^S r) - l(l+1) K(nl) \frac{j_l(k_{nl}^S r)}{k_{nl}^S r} + \right.$$

$$\left. + D(nl) \frac{1}{q_{nl}^S} \frac{d}{dr} y_l(q_{nl}^S r) - l(l+1) \tilde{D}(nl) \frac{y_l(k_{nl}^S r)}{k_{nl}^S r} \right]$$

$$B_{nl}(r) = C(nl) \left[\frac{j_l(q_{nl}^S r)}{q_{nl}^S r} - K(nl) \frac{1}{k_{nl}^S r} \frac{d}{dr} \left\{ r j_l(k_{nl}^S r) \right\} + \right.$$

$$\left. + D(nl) \frac{y_l(q_{nl}^S r)}{q_{nl}^S r} - \tilde{D}(nl) \frac{1}{k_{nl}^S r} \frac{d}{dr} \left\{ r y_l(k_{nl}^S r) \right\} \right]$$

$$(2.11b)$$

Here k_{nl}^SR and q_{nl}^SR are dimensionless eigenvalues, and they are the solution to a rather complicated algebraic equation for the frequencies $\omega = \omega_{nl}$ in (2.1)—see [5] for details. In (2.11a) and (2.11b) we have set

$$K(nl) \equiv \frac{C_{t}q_{nl}^{S}}{C_{l}k_{nl}^{S}}, \qquad D(nl) \equiv \frac{q_{nl}^{S}}{k_{nl}^{S}}E, \qquad \tilde{D}(nl) \equiv \frac{C_{t}Fq_{nl}^{S}}{C_{l}k_{nl}^{S}}$$
 (2.12)

and introduced the normalisation constant C(nl), which is fixed by the orthogonality properties

$$\int_{V} (\mathbf{s}_{n'l'm'}^{S})^* \cdot (\mathbf{s}_{nlm}^{S}) \varrho_0 d^3 x = M \delta_{nn'} \delta_{ll'} \delta_{mm'}$$

$$\tag{2.13}$$

where M is the mass of the hollow sphere:

$$M = \frac{4\pi}{3} \, \varrho_0 R^3 \, (1 - \varsigma^3) \;, \qquad \varsigma \equiv \frac{a}{R} \le 1$$
 (2.14)

Equation (2.13) fixes the value of C(nl) through the radial integral

$$\int_{\varsigma R}^{R} \left[A_{nl}^{2}(r) + l(l+1) B_{nl}^{2}(r) \right] r^{2} dr = \frac{4\pi}{3} \varrho_{0} (1 - \varsigma^{3}) R^{3}$$
(2.15)

as can be easily verified by suitable manipulation of (2.10) and the well known properties of angular momentum operators and spherical harmonics. We shall later specify the values of the different parameters appearing in the above expressions as required in each particular case which will in due course be considered.

III. ABSORPTION CROSS SECTIONS

As seen in reference [3], a scalar–tensor theory of GWs such as JBD predicts the excitation of the sphere's monopole modes as well as the m=0 quadrupole modes. In order to calculate the energy absorbed by the detector according to that theory it is necessary to calculate the energy deposited by the wave in those modes, and this in turn requires that we solve the elasticity equation with the GW driving term included in its right hand side. The result of such calculation was presented in full generality in reference [3], and is directly applicable here because the structure of the oscillation eigenmodes of a hollow sphere is equal to that of the massive sphere —only the explicit form of the wavefunctions needs to be changed. We thus have

$$E_{\text{osc}}(\omega_{nl}) = \frac{1}{2} M b_{nl}^2 \sum_{m=-l}^{l} |G^{(lm)}(\omega_{nl})|^2$$
(3.1)

where $G^{(lm)}(\omega_{nl})$ is the Fourier amplitude of the corresponding incoming GW mode, and

$$b_{n0} = -\frac{\varrho_0}{M} \int_a^R A_{n0}(r) r^3 dr$$
 (3.2a)

$$b_{n2} = -\frac{\varrho_0}{M} \int_a^R \left[A_{n2}(r) + 3B_{n2}(r) \right] r^3 dr$$
 (3.2b)

for monopole and quadrupole modes, respectively, and $A_{nl}(r)$ and $B_{nl}(r)$ are given by (2.11). Explicit calculation yields

$$\frac{b_{n0}}{R} = \frac{3}{4\pi} \frac{C(n0)}{1 - \varsigma^3} \left[\Lambda(R) - \varsigma^3 \Lambda(a) \right]$$
(3.3a)

$$\frac{b_{n2}}{R} = \frac{3}{4\pi} \frac{C(n2)}{1-\varsigma^3} \left[\Sigma(R) - \varsigma^3 \Sigma(a) \right]$$
(3.3b)

with

$$\Lambda(z) \equiv \frac{j_2(q_{n0}z)}{q_{n0}R} + D(n0) \frac{y_2(q_{n0}z)}{q_{n0}R}$$
(3.4a)

$$\Sigma(z) \equiv \frac{j_2(q_{n2}z)}{q_{n2}R} - 3K(n2)\frac{j_2(k_{n2}z)}{k_{n2}R} + D(n2)\frac{y_2(q_{n2}z)}{q_{n2}R} - 3\tilde{D}(n2)\frac{y_2(k_{n2}z)}{k_{n2}R}$$
(3.4b)

The absorption cross section, defined as the ratio of the absorbed energy to the incoming flux, can be calculated thanks to an optical theorem, as proved e.g. by Weinberg [14]. According to that theorem, the absorption cross section for a signal of frequency ω close to ω_N , say, the frequency of the detector mode excited by the incoming GW, is given by the expression

$$\sigma(\omega) = \frac{10 \,\pi \eta c^2}{\omega^2} \, \frac{\Gamma^2 / 4}{(\omega - \omega_N)^2 + \Gamma^2 / 4} \tag{3.5}$$

where Γ is the *linewitdh* of the mode —which can be arbitrarily small, as assumed in the previous section—, and η is the dimensionless ratio

$$\eta = \frac{\Gamma_{\text{grav}}}{\Gamma} = \frac{1}{\Gamma} \frac{P_{GW}}{E_{\text{osc}}} \tag{3.6}$$

where P_{GW} is the energy re-emitted by the detector in the form of GWs as a consequence of its being set to oscillate by the incoming signal.

It is now expedient to split up the energy emitted by the oscillating hollow sphere into two pieces:

$$P_{GW} = P_{\text{pure tensor}} + P_{\text{scalar-tensor}} \tag{3.7}$$

where $P_{\text{pure tensor}}$ is given by General Relativity, and contains only the usual + and \times amplitudes, while $P_{\text{scalar-tensor}}$ is an added term which is only predicted by Jordan-Brans-Dicke theory, and has contributions from a scalar amplitude plus the m = 0 quadrupole amplitude [3,2]. These terms are the following:

$$P_{\text{pure tensor}} = \frac{2G\omega^6}{5c^5} \left[\left| Q_{kk}(\omega) \right|^2 - \frac{1}{3} Q_{ij}^*(\omega) Q_{ij}(\omega) \right]$$
(3.8)

and

$$P_{\text{scalar-tensor}} = \frac{2G\omega^6}{5c^5(2\Omega_{BD} + 3)} \left[|Q_{kk}(\omega)|^2 + \frac{1}{3} Q_{ij}^*(\omega) Q_{ij}(\omega) \right]$$
(3.9)

where $Q_{ij}(\omega)$ is the quadrupole moment of the hollow sphere:

$$Q_{ij}(\omega) = \int_{\text{Antenna}} x_i x_j \, \varrho(\mathbf{x}, \omega) \, d^3 x \tag{3.10}$$

and Ω_{BD} is Brans–Dicke's parameter.

We shall omit any further reference to the pure tensor interaction, as it was rather comprehensively discussed in reference [2]. We thus concentrate in the sequel in the scalar tensor term.

IV. SCALAR-TENSOR CROSS SECTIONS

Explicit calculation shows that $P_{\text{scalar-tensor}}$ is made up of two contributions:

$$P_{\text{scalar-tensor}} = P_{00} + P_{20} \tag{4.1}$$

where P_{00} is the scalar, or monopole contribution to the emitted power, while P_{20} comes from the central quadrupole mode which, as discussed in [2] and [3], is excited together with monopole in JBD theory. One must however recall that monopole and quadrupole modes of the sphere happen at different frequencies, so that cross sections for them only make sense if defined separately. More precisely,

$$\sigma_{n0}(\omega) = \frac{10\pi \,\eta_{n0} \,c^2}{\omega^2} \, \frac{\Gamma_{n0}^2 / 4}{(\omega - \omega_{n0})^2 + \Gamma_{n0}^2 / 4} \tag{4.2a}$$

$$\sigma_{n2}(\omega) = \frac{10\pi \,\eta_{n2} \,c^2}{\omega^2} \, \frac{\Gamma_{n2}^2 / 4}{(\omega - \omega_{n2})^2 + \Gamma_{n2}^2 / 4} \tag{4.2b}$$

where η_{n0} and η_{n2} are defined like in (3.6), with all terms referring to the corresponding modes. After some algebra one finds that

TABLE I. Eigenvalues k_n^R , relative weights D(n0) and H_n coefficients for a hollow sphere with Poisson ratio $\sigma_P = 1/3$. Values are given for a few different thickness parameters ς .

ς	n	$k_{n0}^S R$	D(n0)	H_n
0.01	1	5.48738	$-1.43328 \cdot 10^{-4}$	0.90929
	1	12.2332	$-1.59636 \cdot 10^{-3}$	0.14194
	2	18.6321	$-5.58961 \cdot 10^{-3}$	0.05926
	4	24.9693	-0.001279	0.03267
0.10	1	5.45410	-0.014218	0.89530
	1	11.9241	-0.151377	0.15048
	2	17.7277	-0.479543	0.04922
	4	23.5416	-0.859885	0.04311
0.15	1	5.37709	-0.045574	0.86076
	2	11.3879	-0.434591	0.17646
	3	17.105	-0.939629	0.05674
	4	23.605	-0.806574	0.05396
0.25	1	5.04842	-0.179999	0.73727
	2	10.6515	-0.960417	0.30532
	3	17.8193	-0.425087	0.04275
	4	25.8063	0.440100	0.06347
0.50	1	3.96914	-0.631169	0.49429
	2	13.2369	0.531684	0.58140
	3	25.4531	0.245321	0.01728
	4	37.9129	0.161117	0.07192
0.75	1	3.26524	-0.901244	0.43070
	2	25.3468	0.188845	0.66284
	3	50.3718	0.093173	0.00341
	4	75.469	0.061981	0.07480
0.90	1	2.98141	-0.963552	0.42043
	2	62.9027	0.067342	0.67689
	3	125.699	0.033573	0.00047
	4	188.519	0.022334	0.07538

$$\sigma_{n0}(\omega) = H_n \frac{GMv_S^2}{(\Omega_{BD} + 2) c^3} \frac{\Gamma_{n0}^2 / 4}{(\omega - \omega_{n0})^2 + \Gamma_{n0}^2 / 4}$$
(4.3a)

$$\sigma_{n2}(\omega) = F_n \frac{GMv_S^2}{(\Omega_{BD} + 2)c^3} \frac{\Gamma_{n2}^2/4}{(\omega - \omega_{n2})^2 + \Gamma_{n2}^2/4}$$
(4.3b)

Here, we have defined the dimensionless quantities

$$H_n = \frac{4\pi^2}{9(1+\sigma_P)} (k_{n0}b_{n0})^2 \tag{4.4a}$$

$$F_n = \frac{8\pi^2}{15(1+\sigma_P)} (k_{n2}b_{n2})^2$$
 (4.4b)

where σ_P represents the sphere material's Poisson ratio (most often very close to a value of 1/3), and the b_{nl} are defined in (3.3); v_S is the speed of sound in the material of the sphere.

In tables I and II we give a few numerical values of the above cross section coefficients.

As already stressed in reference [5], one of the main advantages of a hollow sphere is that it enables to reach good sensitivities at lower frequencies than a solid sphere. For example, a hollow sphere of the same material and mass as a solid one ($\varsigma = 0$) has eigenfrequencies which are smaller by

$$\omega_{nl}(\varsigma) = \omega_{nl}(\varsigma = 0) (1 - \varsigma^3)^{1/3}$$
(4.5)

for any mode indices n and l. We now consider the detectability of JBD GW waves coming from several interesting sources with a hollow sphere.

TABLE II. Eigenvalues k_{n2}^R , relative weights K(n2), D(n2), $\tilde{D}(n2)$ and F_n coefficients for a hollow sphere with Poisson ratio $\sigma_P = 1/3$. Values are given for a few different thickness parameters ς .

ς	n	$k_{n2}^S R$	K(n2)	D(n2)	$ ilde{D}(n2)$	F_n
0.10	1	2.63836	0.855799	0.000395	-0.003142	2.94602
	2	5.07358	0.751837	0.002351	-0.018451	1.16934
	3	10.96090	0.476073	0.009821	-0.071685	0.02207
0.15	1	2.61161	0.796019	0.001174	-0.009288	2.86913
	2	5.02815	0.723984	0.007028	-0.053849	1.24153
	3	8.25809	-2.010150	-0.094986	0.672786	0.08113
0.25	1	2.49122	0.606536	0.003210	-0.02494	2.55218
	2	4.91223	0.647204	0.019483	-0.13867	1.55022
	3	8.24282	-1.984426	-0.126671	0.67506	0.05325
	4	10.97725	0.432548	-0.012194	0.02236	0.03503
0.50	1	1.94340	0.300212	0.003041	-0.02268	1.61978
	2	5.06453	0.745258	0.005133	-0.02889	2.29572
	3	10.11189	1.795862	-1.697480	2.98276	0.19707
	4	15.91970	-1.632550	-1.965780	-0.30953	0.17108
0.75	1	1.44965	0.225040	0.001376	-0.01017	1.15291
	2	5.21599	0.910998	-0.197532	0.40944	1.82276
	3	13.93290	0.243382	0.748219	-3.20130	1.08952
	4	23.76319	0.550278	-0.230203	-0.81767	0.08114
0.90	1	1.26565	0.213082	0.001019	-0.00755	1.03864
	2	4.97703	0.939420	-0.323067	0.52279	1.54106
	3	31.86429	6.012680	-0.259533	4.05274	1.46486
	4	61.29948	0.205362	-0.673148	-1.04369	0.13470

V. DETECTABILITY OF JBD SIGNALS

The values of the coefficients F_n and H_n , together with the expressions (4.2) for the cross sections of the hollow sphere, can be used to estimate the detectability of typical GW signals. In this section we report such estimates in terms of the maximum distances at which a coalescing compact binary system and a gravitational collapse event give signal-to-noise ratio equal to one in the detectors. We consider these in turn.

A. Binary systems

We consider as a source of GWs a binary system formed by two neutron stars, each of them with a mass of $m_1 = m_2 = 1.4 \, M_{\odot}$. In the inspiral phase, the system emits a waveform of increasing amplitude and frequency (a "chirp") that can sweep up to the kHz range of frequency. The *chirp mass*, which is the parameter that determines the frequency sweep rate of the chirp signal, corresponding to this system is $M_c \equiv (m_1 m_2)^{3/5} \, (m_1 + m_2)^{-1/5} = 1.22 \, M_{\odot}$, and $\nu_{[5 \text{ cycles}]} = 1270 \, \text{Hz}^1$. Repeating the analysis carried on in section five of [16] we find that the distance at which the signal-to-noise ratio, for a *quantum limited* detector, is equal to one is given by

$$r(\omega_{n0}) = \left[\frac{5 \cdot 2^{1/3}}{32} \frac{1}{(\Omega_{BD} + 2)(12\Omega_{BD} + 19)} \frac{G^{5/3} M_c^{5/3}}{c^3} \frac{M v_S^2}{\hbar \omega_{n0}^{4/3}} H_n \right]^{1/2}$$
(5.1a)

$$r(\omega_{n2}) = \left[\frac{5 \cdot 2^{1/3}}{192} \frac{1}{(\Omega_{BD} + 2)(12\Omega_{BD} + 19)} \frac{G^{5/3} M_c^{5/3}}{c^3} \frac{M v_S^2}{\hbar \omega_{n2}^{4/3}} F_n \right]^{1/2}$$
(5.1b)

¹ The frequency $\nu_{[5 \text{ cycles}]}$ is the one the system has when it is 5 cycles away from coalescence. It is considered that beyond this frequency disturbing effects distort the simple picture of a clean Newtonian binary system —see [15] for further references.

TABLE III. Eigenfrequencies, sizes and distances at which coalescing binaries can be seen by monitoring of their emitted JBD GWs. Figures correspond to a 200 ton CuAl hollow sphere.

ς	Φ (m)	$ u_{10}(\mathrm{Hz})$	$\nu_{12} \; (\mathrm{Hz})$	$r(\nu_{10}) \; (\mathrm{kpc})$	$r(\nu_{12}) \; (\mathrm{kpc})$
0.25	3.72	1243	613	92	50
0.50	3.88	937	459	91	48
0.75	4.46	671	298	106	54
0.90	5.74	476	202	131	67

TABLE IV. Eigenfrequencies, sizes and distances at which coalescing binaries can be seen by monitoring of their emitted JBD GWs. Figures correspond to a 6 metres external diameter CuAl hollow sphere.

ς	M (ton)	$ u_{10}(\mathrm{Hz}) $	$ u_{12}(\mathrm{Hz}) $	$r(\nu_{10}) \; (\mathrm{kpc})$	$r(\nu_{12}) \text{ (kpc)}$
0.25	832	770	380	257	140
0.50	740	609	296	233	124
0.75	489	498	221	202	104
0.90	230	455	193	145	74

For a CuAL sphere, the speed of sound is $v_S = 4700$ m/sec. We report in table III the maximum distances at which a JBD binary can be seen with a 200 ton hollow spherical detector, including the size of the sphere (diameter and thickness factor). The Brans-Dicke parameter Ω_{BD} has been given a value of 600. This high value has as a consequence that only relatively nearby binaries can be scrutinised by means of their scalar radiation of GWs. A slight improvement in sensitivity is appreciated as the diameter increases in a fixed mass detector.

B. Gravitational collapse

The signal associated to a gravitational collapse has recently been modeled, within JBD theory, as a short pulse of amplitude b, whose value can be estimated as [2]:

$$b \simeq 10^{-23} \left(\frac{500}{\omega_{BD}}\right) \left(\frac{M}{M_{\odot}}\right) \left(\frac{10 \, Mpc}{r}\right) \tag{5.2}$$

The minimum value of the Fourier transform of the amplitude of the scalar wave, for a quantum limited detector at unit signal-to-noise ratio, is given by

$$|b(\omega_{nl})|_{\min} = \left(\frac{4\hbar}{Mv_S^2 \omega_{nl} K_n}\right)^{1/2} \tag{5.3}$$

where $K_n = 2H_n$ for the mode with l = 0 and $K_n = F_n/3$ for the mode with l = 2, m = 0.

The duration of the impulse, $\tau \approx 1/f_c$, is much shorter than the decay time of the nl mode, so that the relationship between b and $b(\omega_{nl})$ is

$$b \approx |b(\omega_{nl})| f_c$$
 at frequency $\omega_{nl} = 2\pi f_c$ (5.4)

so that the minimum scalar wave amplitude detectable is

$$|b|_{\min} \approx \left(\frac{4\hbar\omega_{nl}}{\pi^2 M v_S^2 K_n}\right)^{1/2} \tag{5.5}$$

Let us now consider a hollow sphere made of molibdenum, for which the speed of sound is as high as $v_S = 5600$ m/sec. For a given detector mass and diameter, equation (5.5) tells us which is the minimum signal detectable with such detector. For example, a solid sphere of 31 tons and 1.8 metres in diameter, is sensitive down to $b_{\min} = 1.5 \cdot 10^{-22}$. Equation (5.2) can then be inverted to find which is the maximum distance at which the source can be identified by the scalar waves it emits. Taking a reasonable value of $\Omega_{BD} = 600$, one finds that $r(\nu_{10}) \approx 0.6$ Mpc.

Like before, we report in tables V and VI the sensitivities of the detector and consequent maximum distance at which the source appears visible to the device for various values of the thickness parameter ς . In table V a constant detector mass of 31 tons has been assumed for all thicknesses, and in table VI a constant outer diameter of 1.8 metres in all cases.

TABLE V. Eigenfrequencies, maximum sensitivities and distances at which a gravitational collapse can be seen by monitoring the scalar GWs it emits. Figures correspond to a 31 ton Mb hollow sphere.

ς	ϕ (m)	$\nu_{10}~({ m Hz})$	$ b _{\min} \ (10^{-22})$	$r(\nu_{10}) \text{ (Mpc)}$
0.00	1.80	3338	1.5	0.6
0.25	1.82	3027	1.65	0.5
0.50	1.88	2304	1.79	0.46
0.75	2.16	1650	1.63	0.51
0.90	2.78	1170	1.39	0.6

TABLE VI. Eigenfrequencies, maximum sensitivities and distances at which a gravitational collapse can be seen by monitoring the scalar GWs it emits. Figures correspond to a 1.8 metres outer diameter Mb hollow sphere.

ς	M (ton)	$\nu_{10}~(\mathrm{Hz})$	$ b _{\min} (10^{-22})$	$r(\nu_{10}) \; (\mathrm{Mpc})$
0.00	31.0	3338	1.5	0.6
0.25	30.52	3062	1.71	0.48
0.50	27.12	2407	1.95	0.42
0.75	17.92	1980	2.34	0.36
0.90	8.4	1808	3.31	0.24

VI. CONCLUSIONS

In this paper we have explored the ability of a hollow sphere to sense the GWs emitted by radiating systems which obey the laws of Jordan-Brans-Dicke theory rather than those of General Relativity. The difference between the predictions of both theories is in the degrees of freedom of the signal: while GR only predicts two polarisation states —the usual + and \times amplitudes—, JBD predicts a scalar amplitude and one more quadrupole amplitude (m=0) in addition to the + and \times amplitudes. We have calculated the generic cross sections of the hollow spherical detectors for the first few monopole and quadrupole harmonics, which are relevant to the detection of JBD waves, then applied the results to the analysis of how efficient the detection of such waves coming from a coalescing compact binary system and a collapsing star, respectively, can be with a hollow spherical GW detector.

In terms of the detector thickness it is seen that sensitivity appears to be, in absolute figures, quite independent of the detector geometry, i.e., the weakest detectable signal in the best experimental circumstances, does not change in order of magnitude over the entire range of possible thickness parameter values. In other words, there is not much difference in sensitivity between a hollow sphere and a massive sphere [2] for this kind of detection problem. The one appreciable difference, however, is that one can reach considerably lower frequencies in the sensitivity range of a hollow sphere than can with a hollow sphere. Not surprisingly, the general facts described in reference [5] also survive in this more general framework of JBD theory.

- [1] C.M. Will, Theory and Experiment in Gravitational Physics (Cambridge University Press, Cambridge, 1993).
- [2] M. Bianchi, M. Brunetti, E. Coccia, F. Fucito, and J.A. Lobo, Phys. Rev. D 57, 4525 (1998).
- [3] J.A. Lobo, *Phys. Rev.* D 52, 591 (1995).
- [4] R.V. Wagoner and H.J. Paik in *Proc. of the Int. Symposium on Experimental Gravitation* (Accademia Nazionale dei Lincei, Rome, 1977).
- [5] E. Coccia, V. Fafone, G. Frossati, J. A. Lobo, and J. A. Ortega, Phys. Rev. D 57, 2051 (1998).
- [6] F.J. Raab (LIGO team), in E. Coccia, G.Pizzella, F.Ronga (eds.), *Gravitational Wave Experiments*, Proceedings of the First Edoardo Amaldi Conference, Frascati 1994 (World Scientific, Singapore, 1995).
- [7] A. Giazotto et al. (VIRGO collaboration), in E. Coccia, G.Pizzella, F.Ronga (eds.), Gravitational Wave Experiments, Proceedings of the First Edoardo Amaldi Conference, Frascati 1994 (World Scientific, Singapore, 1995).
- [8] P. Jordan, Z. Phys., 157, 112 (1959).
- [9] C. Brans and R. H. Dicke, Phys. Rev. 124, 925 (1961).
- [10] A.E.H. Love, A Treatise on the Mathematical Theory of Elasticity, Dover 1944.
- [11] L. D. Landau and E. M. Lifshitz, Theory of Elasticity, Pergamon 1970.
- [12] M. Abramowitz and I.A. Stegun, Handbook of Mathematical Functions, Dover 1972.

- [13] M. Bianchi, E. Coccia, C. N. Colacino, V. Fafone, and F. Fucito, Class. and Quantum Grav. 13, 2865 (1996).
- [14] S. Weinberg, Gravitation and Cosmology, Wiley & sons, New York 1972.
- [15] E. Coccia and V. Fafone, Phys. Lett. A 213, 16 (1996).
- [16] M. Brunetti, E. Coccia, V. Fafone and F. Fucito, *Phys. Rev.* D 59, 044027 (1999).